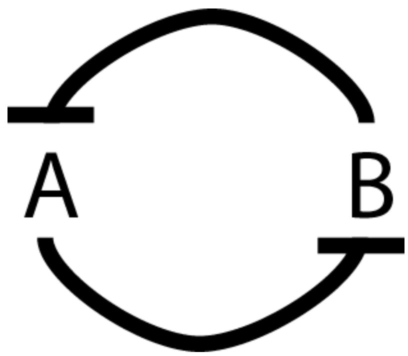
Quantitative Understanding in Biology Module IV: Differential Equations Computer Laboratory

Dynamic Systems and Bifurcation

Consider the genetic switch model we worked on in class *Reference: (Gardner, Cantor and Collins, Nature, 2000)….*



$$\frac{dx}{dt} = -x + \frac{\alpha}{1 + y^n}$$
$$\frac{dy}{dt} = -y + \frac{\alpha}{1 + x^n}$$

We saw that when the Hill parameter, n, is two and basal production, α, is three, we find two stable steady states and one unstable steady state.

Show that for (n, α) = (3, 1), there is only one equilibrium point. Find it, and investigate its stability.

Keeping n fixed at 2, find the value of α at which the number of equilibrium points changes.

This point in parameter space, that is (n, α) space, is called a bifurcation point.

Prepare a plot showing how the steady-state points move as α varies while n is held fixed at n=2.

Hint: This kind of bifurcation is called a 'pitchfork' bifurcation.

Challenge (optional): **Prepare a map showing where in (n, α) space the system has one equilibrium point and where it has three equilibrium points. If you were trying to engineer a biological switch, where would you want to be in this space?**

Numerical Simulation with External Stimuli

Run a numerical simulation (using MATLAB's ode45 function), for the case where n = 2, α = 3 and starting concentrations of A and B at 2 and 1, respectively. Plot your results.

You should see convergence to expected equilibrium values in about 25 time units.

Add a short but significant influx of B at t=50 into your model. Experiment numerically to determine how much and how quickly you need to perturb the system to get the switch to 'flip' state.

Hint: You need to add terms into your differential equations to model the addition of material into the system. One of the parameters available to you is the absolute time of the simulation; you'll need to use that.

