Population Dynamics in the Presence of Infectious Diseases

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FINAL PROJECT

QBIO, Spring 2009

INTRODUCTION

INFECTIOUS DISEASES:

- Epidemic
- Endemic

FACTORS:

- Sanitation and good water supply
- Human behavior
- Antibiotics and vaccination programs
- Agents that adapt and evolve
- Climate change

COMPARTMENTS	PARAMETERS
• M= Passively Immune • S= Susceptible • E=Exposed • I=Infective • R=Recovered • T=Treated • V=Vaccinated	• β=contact rate • $1/\delta$=period of passive immunity • $1/\varepsilon$= latent period • $1/\gamma$=infectious period

Contact Rate

- Contact Rate β is the number of 'adequate contacts' that a person makes per time
- For example if each person makes 10 'adequate contacts' on average per year, the contact rate would be 10 per year.
- If the infective fraction of the population is i=1/3, 1/3 of these contacts would be with the infectives.
- The susceptible population S=3000 would then have 3000*10/3=100 adequate contacts with the infectives within a year.
- This is the number of new cases per time.

Simplest Model: SIR for an Epidemic

S=Susceptible

I=Infected

R=Recovered

β=contact rate

1/γ=infectious disease duration

- Population N

1. $dS/dt=-\beta IS/N$
2. $dI/dt= \beta IS/N- \gamma I$
3. $dR/dt= \gamma I$

- Dividing them by N

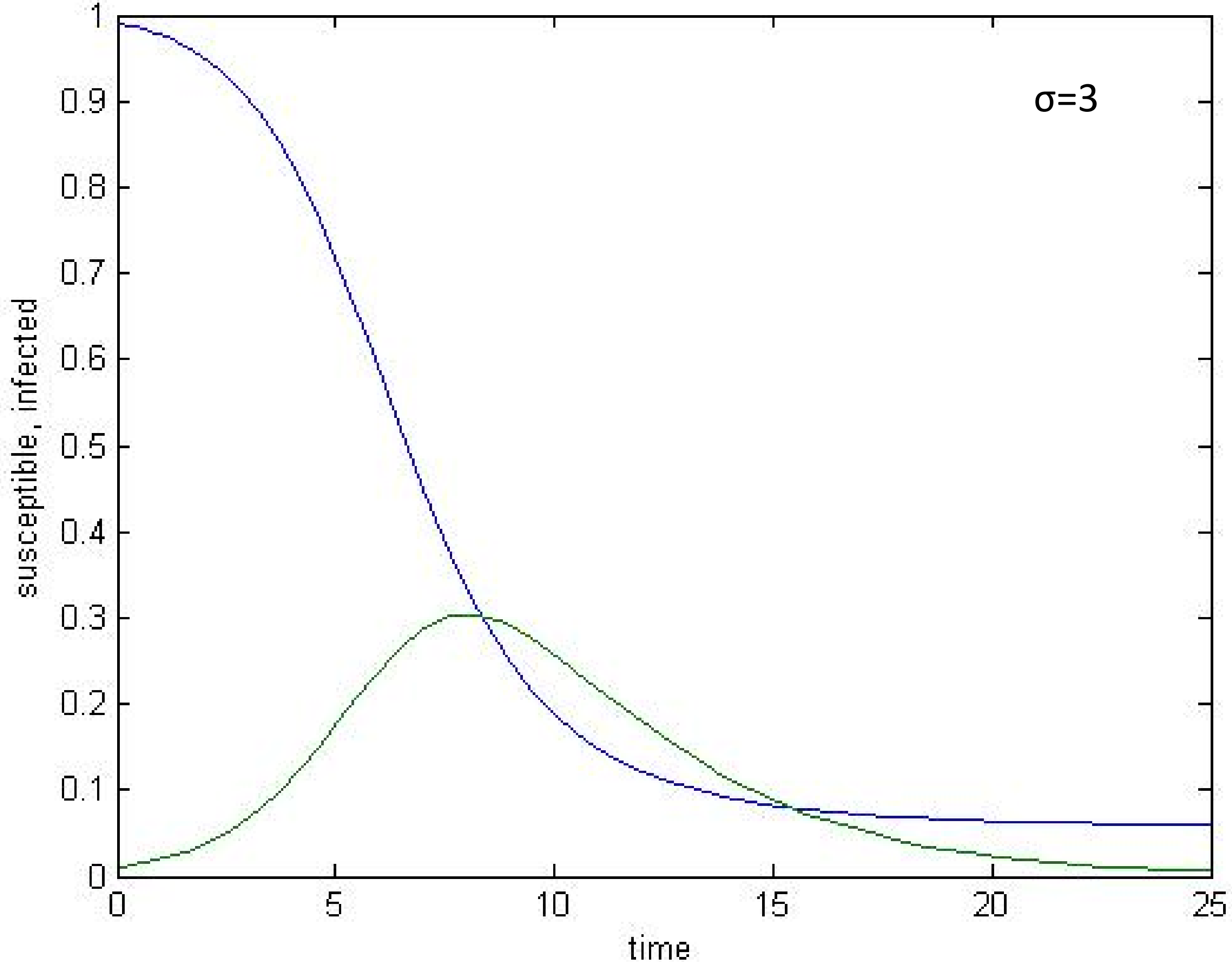
1. $ds/dt=-\beta is$
2. $di/dt= \beta is- \gamma i$
3. $dr/dt= \gamma i$

Adequate Contact Number (σ) = Contact Rate * Disease Duration= β/γ

Replacement Number *R*(t)= Adequate Contact Number* Fraction of Susceptibles= *σs(t)*

Susceptible:	0.99	$1/\sigma$	$s\infty$
Infective:	0.01	imax	0

At the peak of i, $R = s\sigma = 1$
As s keeps decreasing
R <1 and therefore i starts to decrease



Contact and Replacement

- Contact number σ is the number of 'adequate contacts' that an infective person makes throughout the infected period = β/γ
- The replacement number *R* is, the number of susceptible people that an infective person makes contact with throughout the infectious period $R = s\sigma$
- When the fraction of susceptible population is more than $1/\sigma$, the replacement number is more than 1, the infection spreads. When it is less than $1/\sigma$, the infection declines.

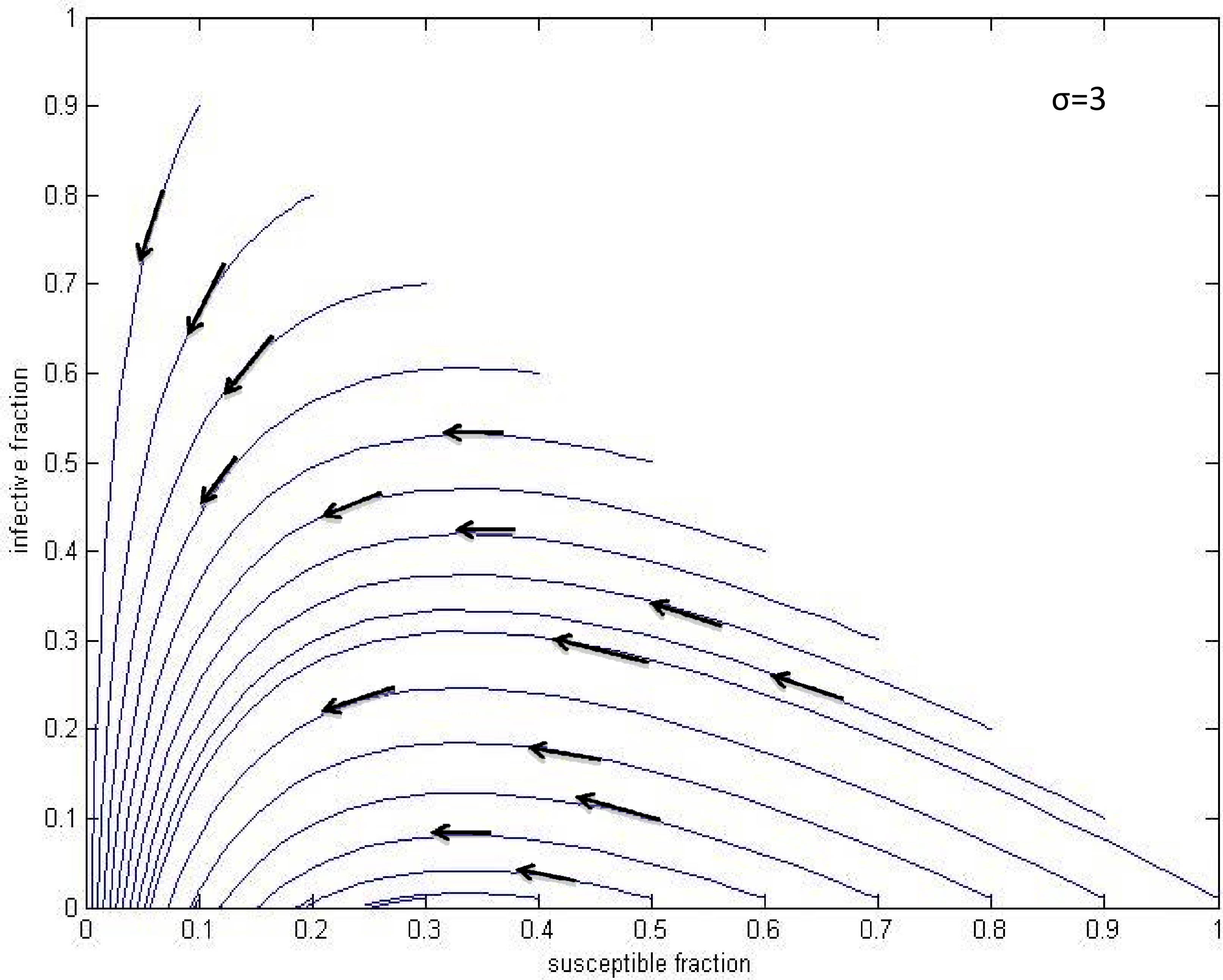
The susceptible function is a decreasing function and the limiting value can be found by:

1. $ds/dt=-\beta is$
2. $di/dt= \beta is- \gamma i$

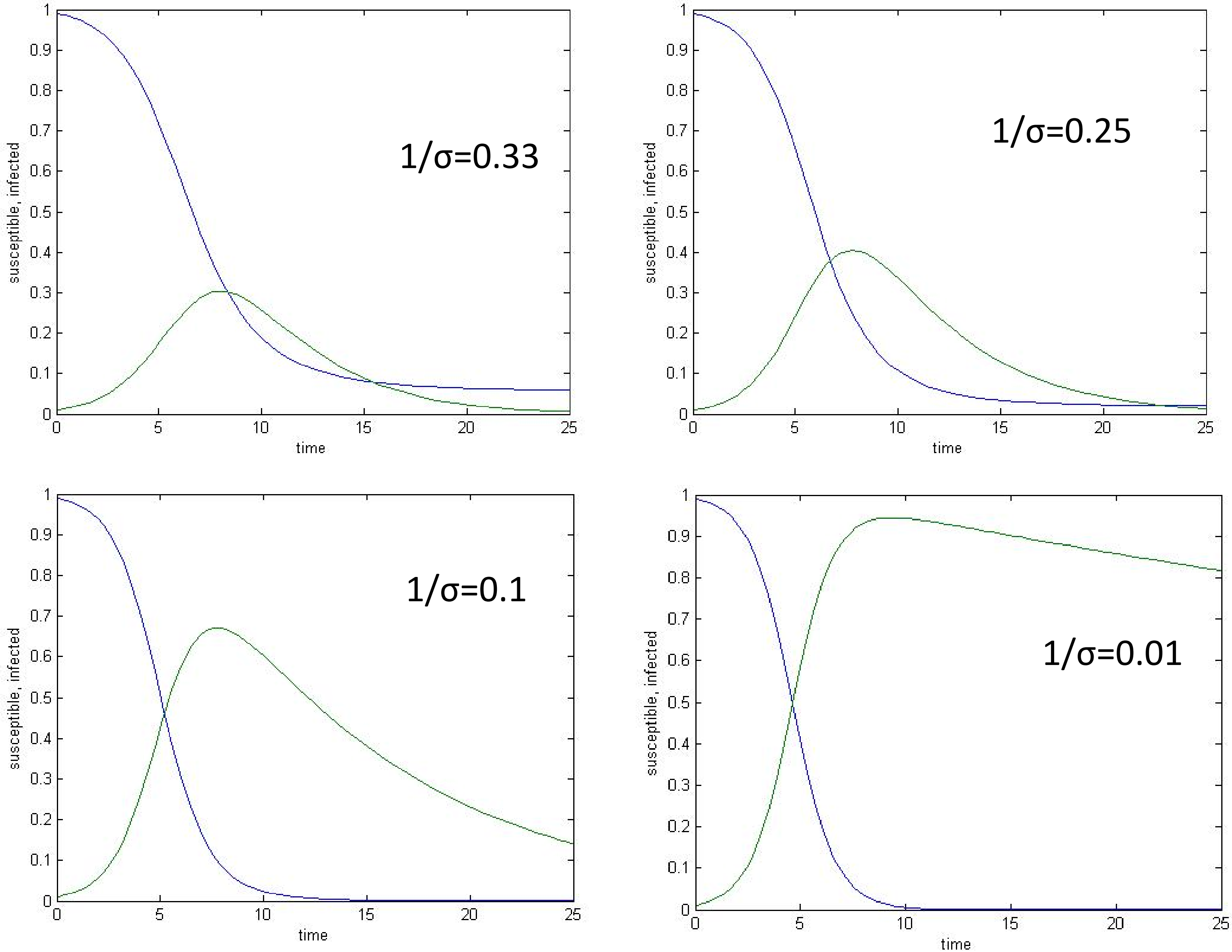
- $di=(-1+(1/\sigma s))ds$
- Integrate from 0 to infinity:
- $i_\infty-i_0=s_0-s_\infty+\ln(s_\infty/s_0)/\sigma$
- $i_0+s_0-s_\infty+\ln(s_\infty/s_0)/\sigma=0$

Calculating backward

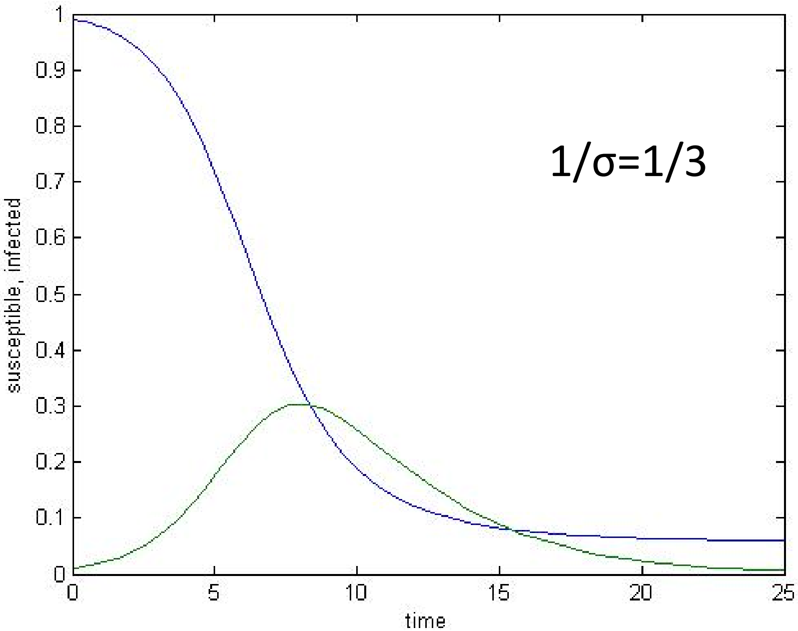
- The equation we had for the limiting value of the susceptible fraction was
- $i_0+s_0-s_\infty+\ln(s_\infty/s_0)/\sigma=0$
- For negligible small initial infective fraction i_0
- $\sigma \approx \ln(s_0/s_\infty) / s_0-s$
- By using data at the beginning and at the end of the epidemic, estimate the contact number

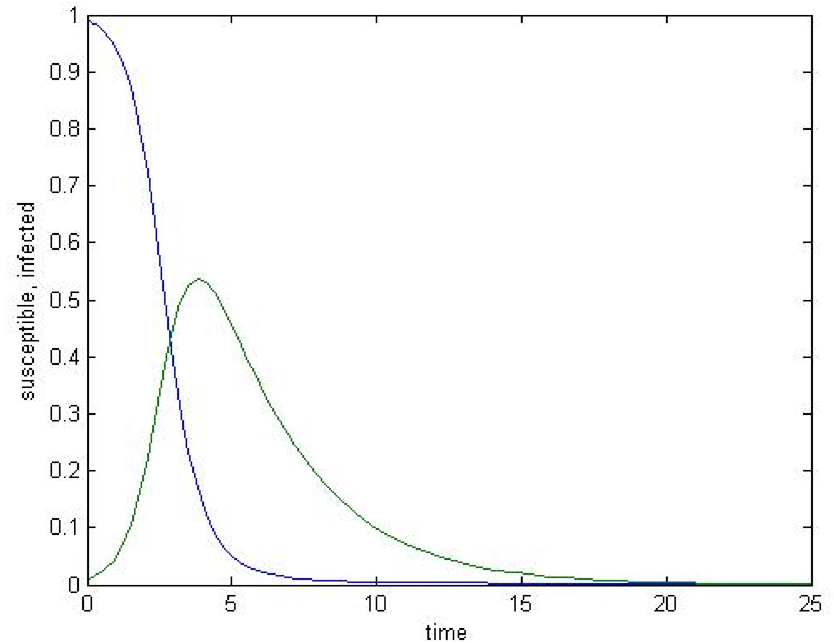

σ=3
infective fraction
susceptible fraction
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Increasing disease period



Or the contact rate



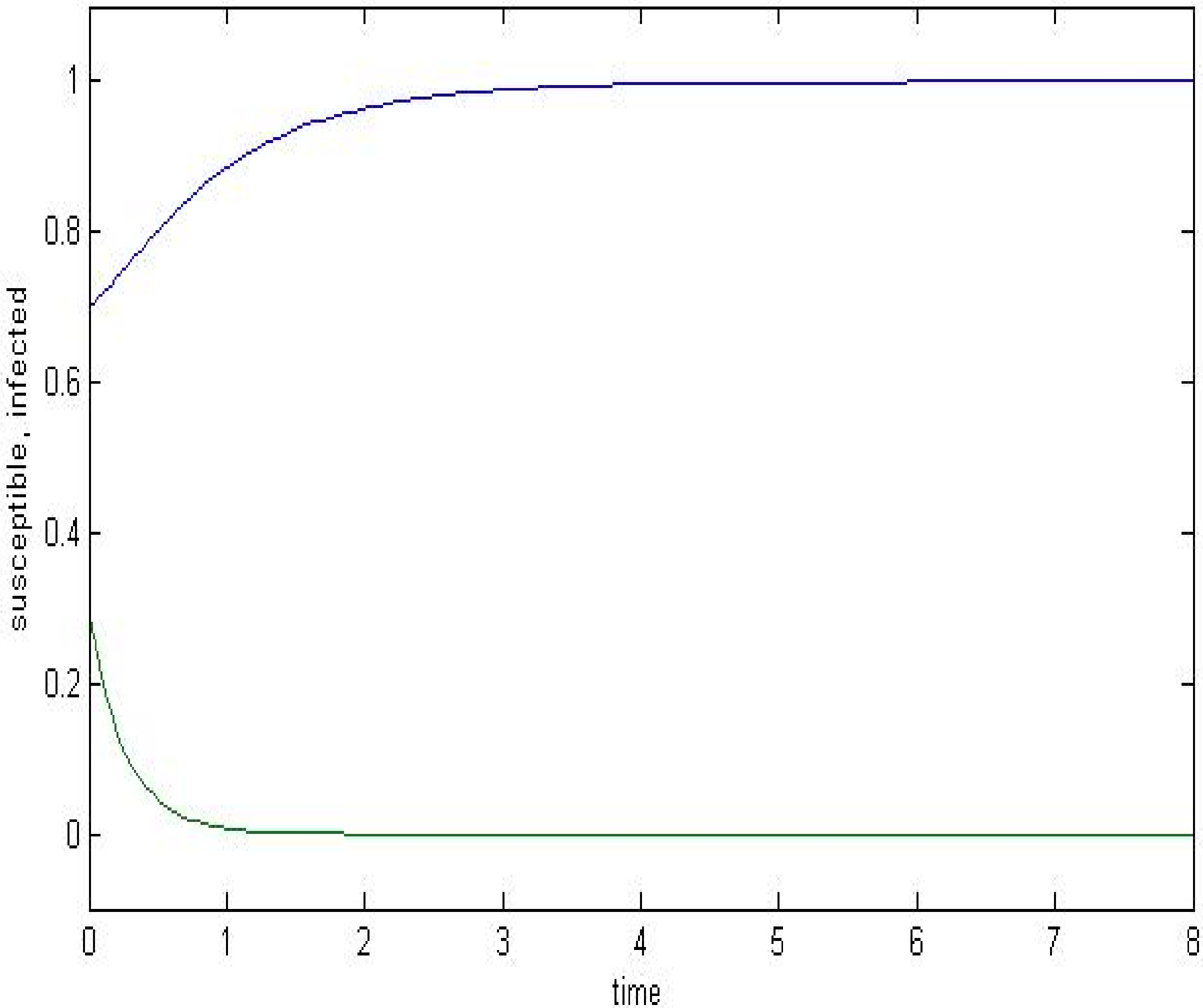


Limiting value of s goes down with increasing contact number

Endemics:
With Vital Birth/Death Dynamics

1. $ds/dt=\mu-\mu s-\beta is$
2. $di/dt= \beta is- \gamma i- \mu i$
3. $dr/dt= \gamma i- \mu r$

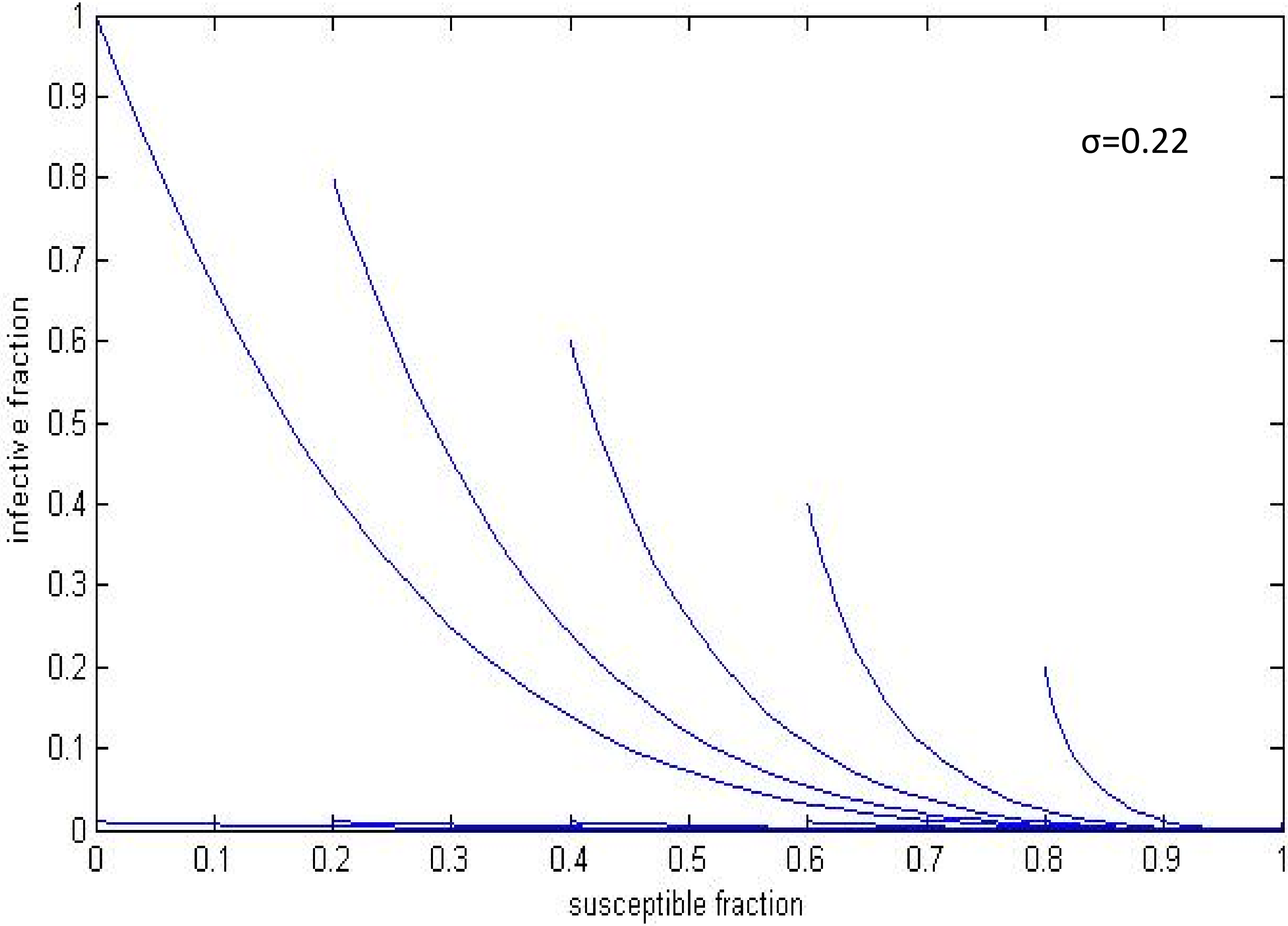
Replacement number $R=s\sigma$,
where the contact number σ is adjusted for death rate: $\sigma=\beta/(\gamma+m)$
Just like the epidemic case, for $R0=s0*\sigma>1$, i starts to go up, s starts to go down. Then when s is low enough, such that R crosses the threshold 1, i starts to go down as well. Unlike the epidemic case, new borns are introduced into the s group and increase the replacement number again such that i starts to go back up. This continues until an equilibrium is reached.



For $\sigma<1$

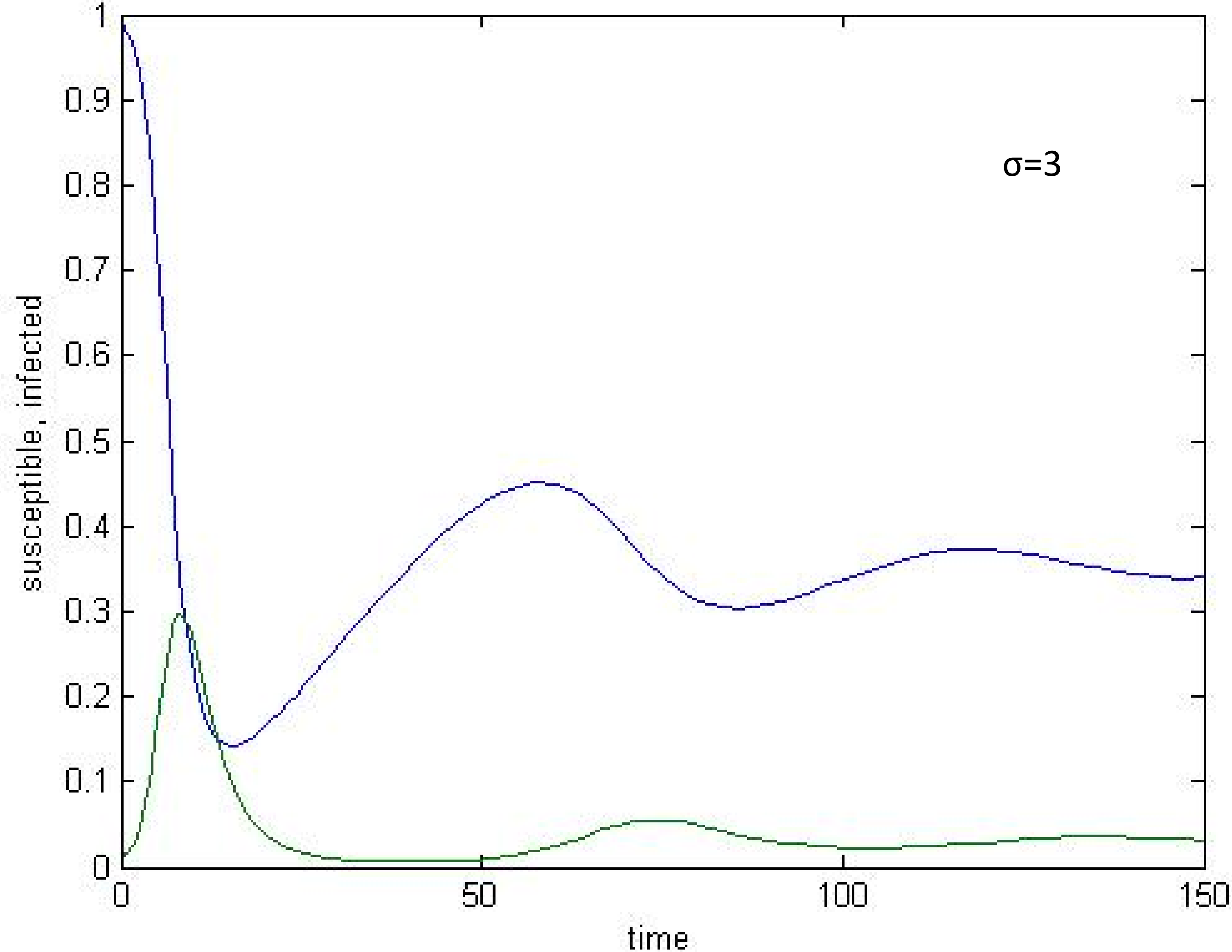
$S_{eq}=1$

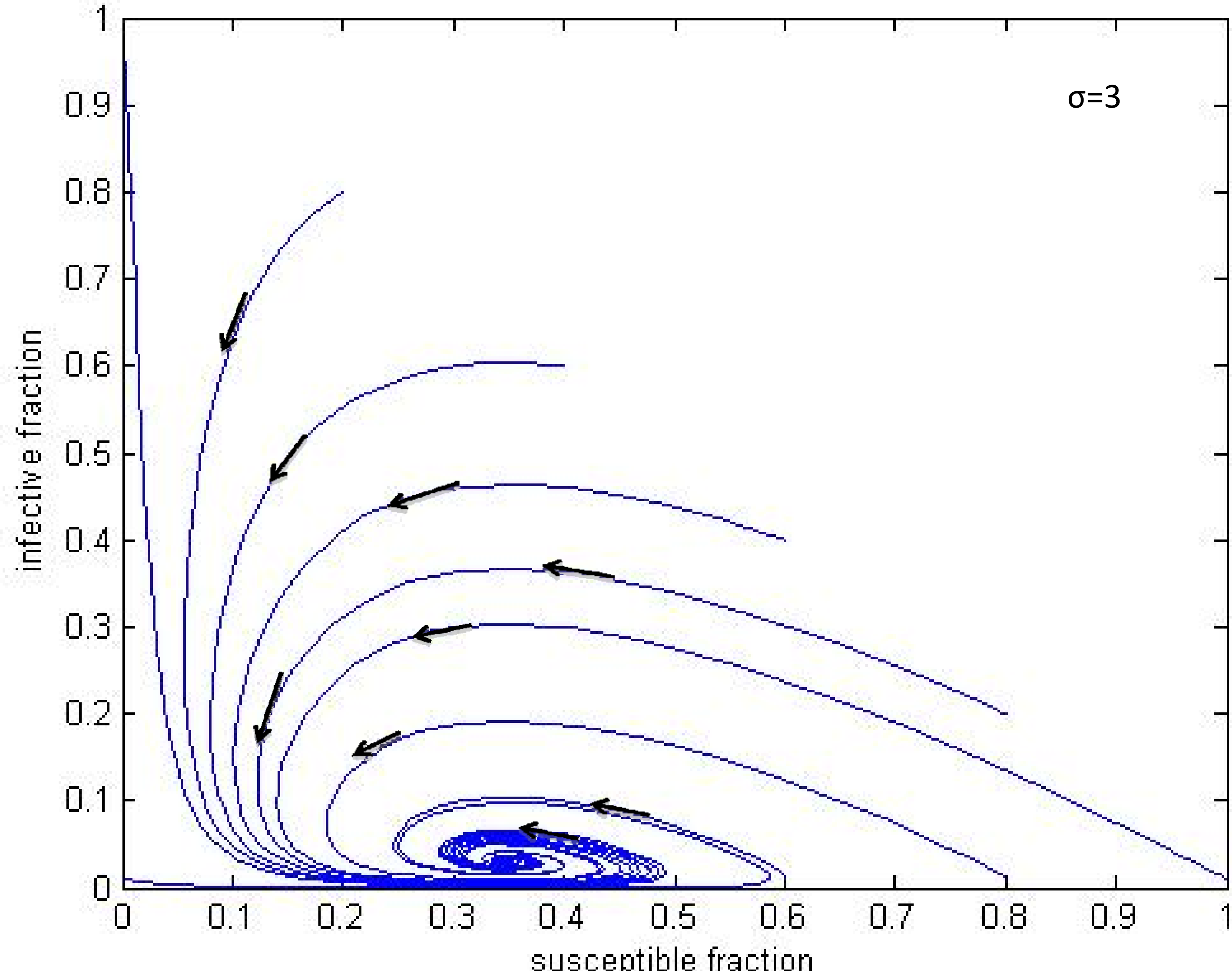
$i_{eq}=0$



σ=0.22
infective fraction
susceptible fraction
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Endemic Equilibrium




σ=3
infective fraction
susceptible fraction
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Endemic Equilibrium

1. $ds/dt=\mu-\mu s-\beta is=0$
2. $di/dt= \beta is- \gamma i- \mu i=0$

→ $s_{eq}= (\gamma+\mu) /\beta=1/\sigma$

→ $i_{eq}= \mu(\sigma-1)/\beta$

Stability at the Equilibrium

- $dx/dt=f(x,y)$
- $dy/dt=g(x,y)$

- $u=x-x_{eq}$
- $z=y-y_{eq}$

At Equilibrium:

$dx/dt=f(x_{eq},y_{eq})=0$

$dy/dt=g(x_{eq},y_{eq})=0$

Near Equilibrium:

$du/dt=f(x,y)\approx f(x_{eq}\cancel{,y_{eq}})+\delta f/\delta x(x_{eq},y_{eq})u+\delta f/\delta y(x_{eq},y_{eq})z$

$dz/dt=g(x,y)\approx g(x_{eq}\cancel{,y_{eq})}+\delta g/\delta x(x_{eq},y_{eq})u+\delta g/dy(x_{eq},y_{eq})z$

Jacobian Matrix

J=

$\delta f/\delta x(xeq,yeq)$	$\delta f/\delta y(xeq,yeq)$
$\delta g/\delta x(xeq,yeq)$	$\delta g/\delta y(xeq,yeq)$

If the dominant eigenvalue is larger than zero, it's unstable
If the dominant eigenvalue is smaller than zero, it's stable

Endemic Jacobian=

$-\beta i_{eq}-\mu$	$-\beta s_{eq}$
βi_{eq}	$-(\gamma+\mu)+\beta s_{eq}$

Endemic Example

- For σ>1
- We plug in the endemic equilibrium solutions and find the eigenvalues
- Note that they need to be negative since the last term is negative for σ>1

$$-\frac{\mu\beta}{2(\gamma+\mu)} \pm \sqrt{\left(\frac{\mu\beta}{2(\gamma+\mu)}\right)^2 - 4\mu(\beta-\gamma-\mu)}$$

Changing Population Sizes

1. $dS/dt = bN - \beta IS/N - dS$
2. $dI/dt = \beta IS/N + (\gamma + d)I$
3. $dR/dt = \gamma I - dR$
4. $d(S+I+R)/dt = q$

Dividing them by N

1. $ds/dt = b - \beta is - ds - qs$
2. $di/dt = \beta is - (\gamma + d + q)i$
3. $dr/dt = \gamma i - dr - qr$
4. $d(s+i+r) = 0$

Birth rate=b

Death rate=d

Size change=q=b-d

$dN/dt = (b-d)N$

$\sigma=\beta/(\gamma+d+q)= \beta/(\gamma+b)$

- As long as average life duration is much longer than the disease duration… with increasing birth rate, i goes up too

1. $ds/dt=b-(d+q)s-\beta is=0$
2. $di/dt= \beta is- (\gamma+d+q)i=0$

→ $s_{eq}=1/\sigma= (\gamma+d+q)/\beta$

→ $i_{eq}= b(\sigma-1)/\beta$

Latent Period

1. $dS/dt = bN - \beta IS/N - dS$
2. $dE/dt = \beta IS/N - (\varepsilon + d)E$
3. $dI/dt = \varepsilon E - (\gamma + d)I$
4. $dR/dt = \gamma I - dR$

Dividing them by N

1. $ds/dt = b - \beta is - (d+q)s$
2. $de/dt = \beta is - (\varepsilon + d + q)e$
3. $di/dt = \varepsilon e - (\gamma + d + q)i$
4. $dr/dt = \gamma i - (d+q)r$

Birth rate=b

Death rate=d

$dN/dt = (b-d)N$

$\sigma = \beta\varepsilon/(\varepsilon + d + q)(\gamma + d + q)$

Treatment Group

1. $dS/dt = bN - \beta IS/N - dS$
2. $dE/dt = \beta IS/N - (\varepsilon + d)E$
3. $dI/dt = \varepsilon E - (\gamma + d + f)I$
4. $dT/dt = fI - (\gamma' + d)T$
5. $dR/dt = (\gamma I + \gamma' T) - dR$

Dividing them by N

1. $ds/dt = b - \beta is - (d+q)s$
2. $de/dt = \beta is - (\varepsilon + d + q)e$
3. $di/dt = \varepsilon e - (\gamma + d + q - f)i$
4. $dt/dt = fi - (\gamma' + d + q)t$
5. $dr/dt = (\gamma i + \gamma' t) - (d+q)r$

Birth rate=b

Death rate=d

$dN/dt = (b-d)N$

Scenario with a Treatment Program

- A new kind of infectious disease emerges in a community with an initial population of 7000 people
- Birth rate = 0.025
- Death rate= 0.015 (life expectancy=67 years)
- On average, latency is about 1 month long
- The infected people suffer from it for 2 years
- The adequate contact rate is 2 per year
- The disease spreads for 5 years
- After 50 years, a drug is developed, which can cure the disease in 3 months
- On average, infected people start taking drugs 2 months after the infection

Without treatment:

200 years after the disease intro:

S:0.2635

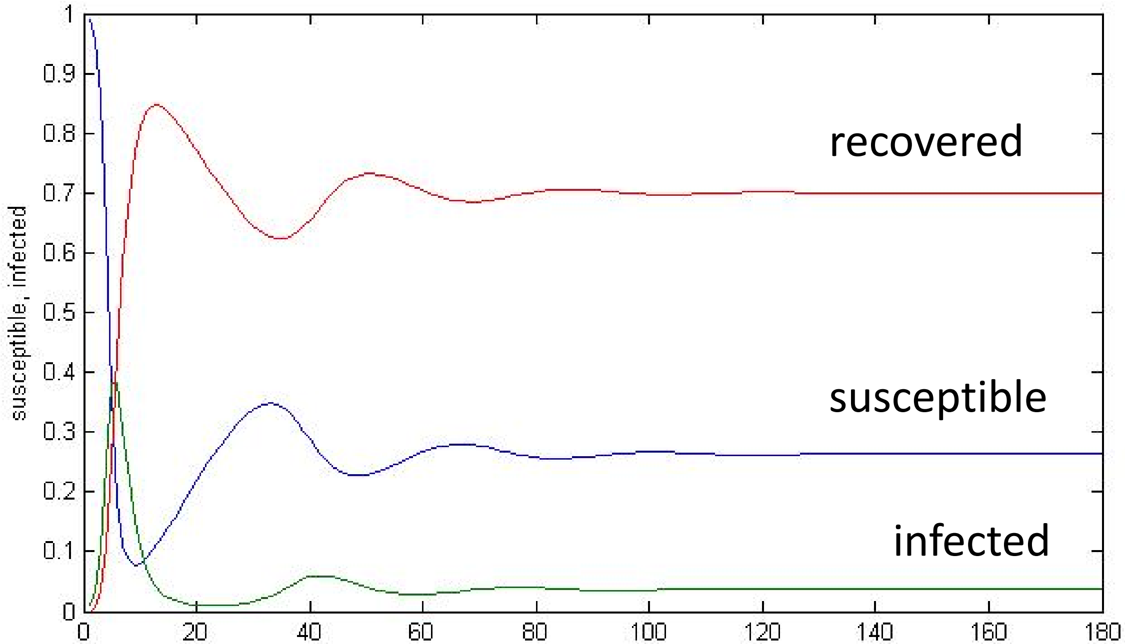
E:0.0018

I:0.0347

T:0

R:0.6999

N:21000



With treatment:

200 years after the disease intro:

S:0.3625

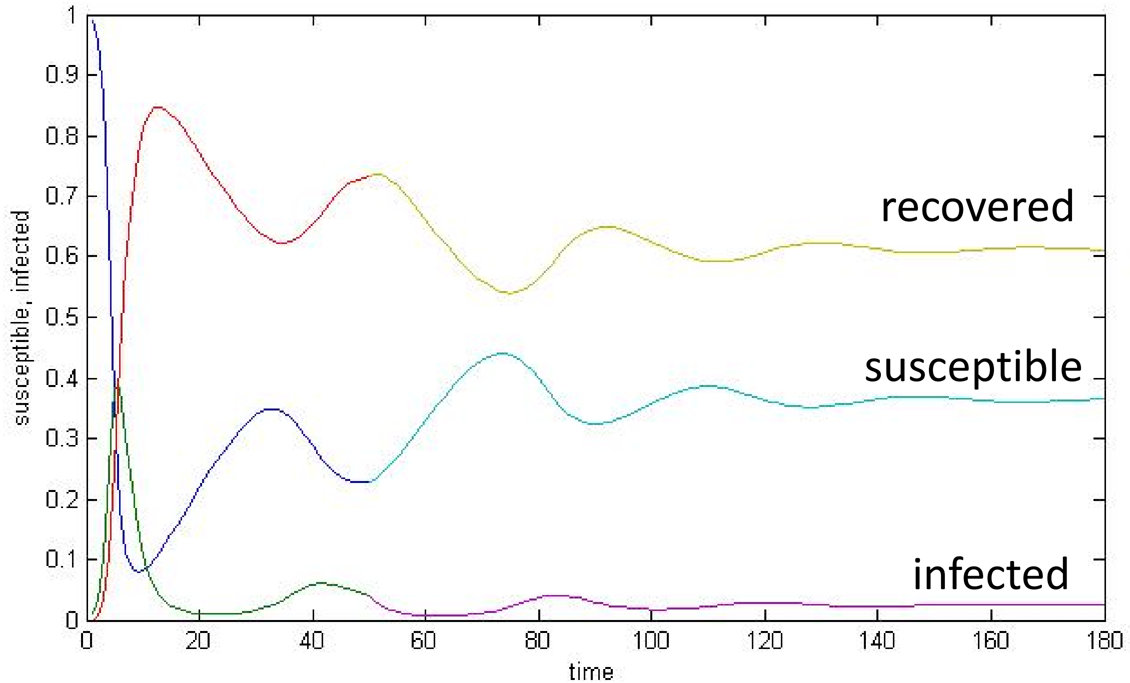
E:0.0016

I:0.0221

T:0.0011

R:0.6127

N:21000



DRUG QUANTIFICATION

$$B = \sum_{i=current\ year}^{i=current\ year+5} N(i)t(i)q(i)d,$$

where B is the total budget needed,

$N(i)$ *is the population*

t is the treated fraction

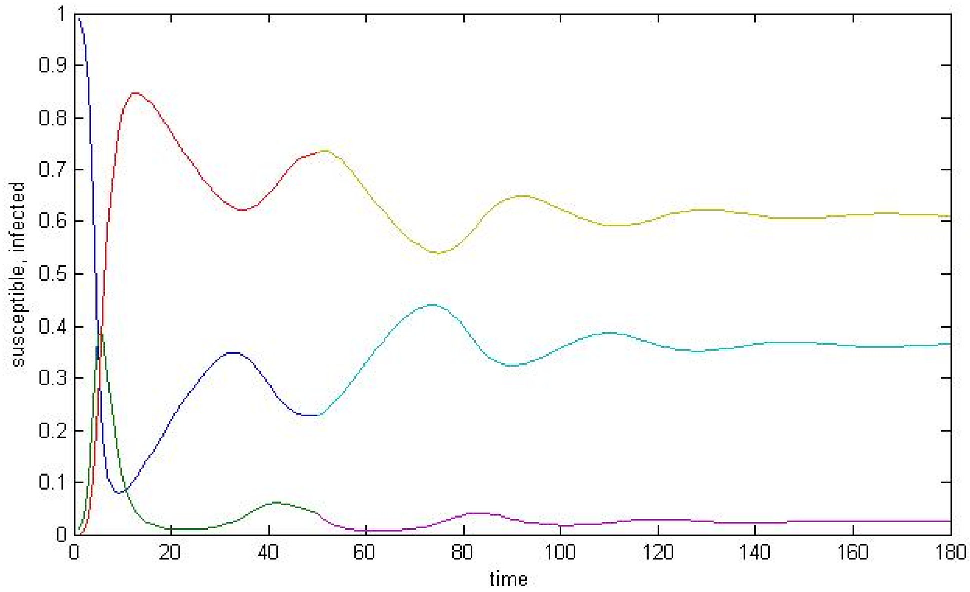
p is the drug price per unit

q is the quantity of drugs needed per person per year

A Scenario with A Treatment Program

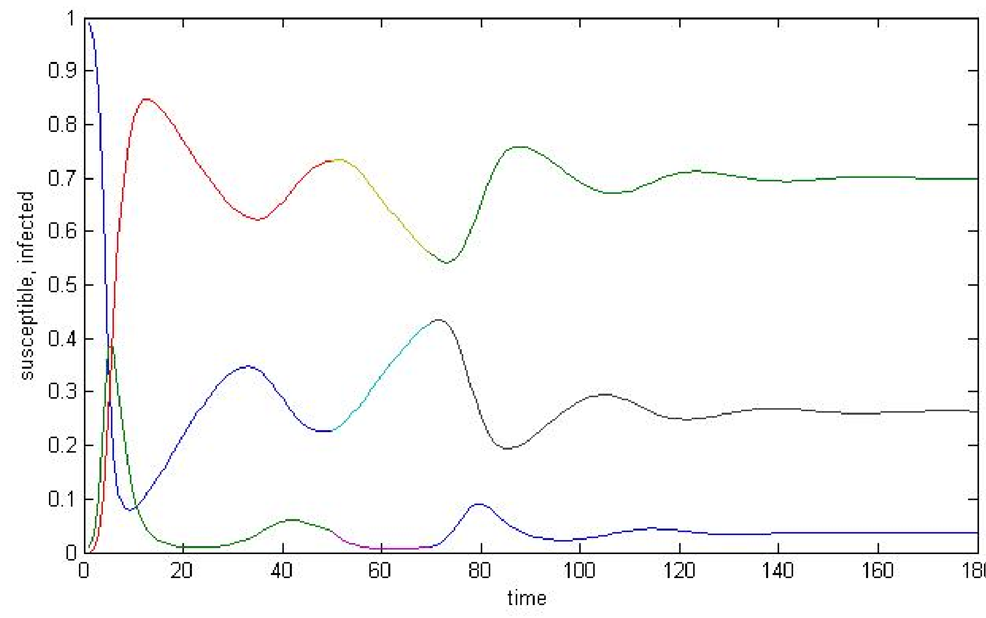
- After yet another 20 years (during the 3rd peak of the infectious fraction- which is the 1st after the drug introduction), the policy makers realize that the budget needed for the drug is increasing.
- Assuming the price for the drug remains constant, what's the predicted budget for the first 5, 10, 15 years after this realization?
- 5 years: 19439
- 10 years: 57513 (additional 38074)

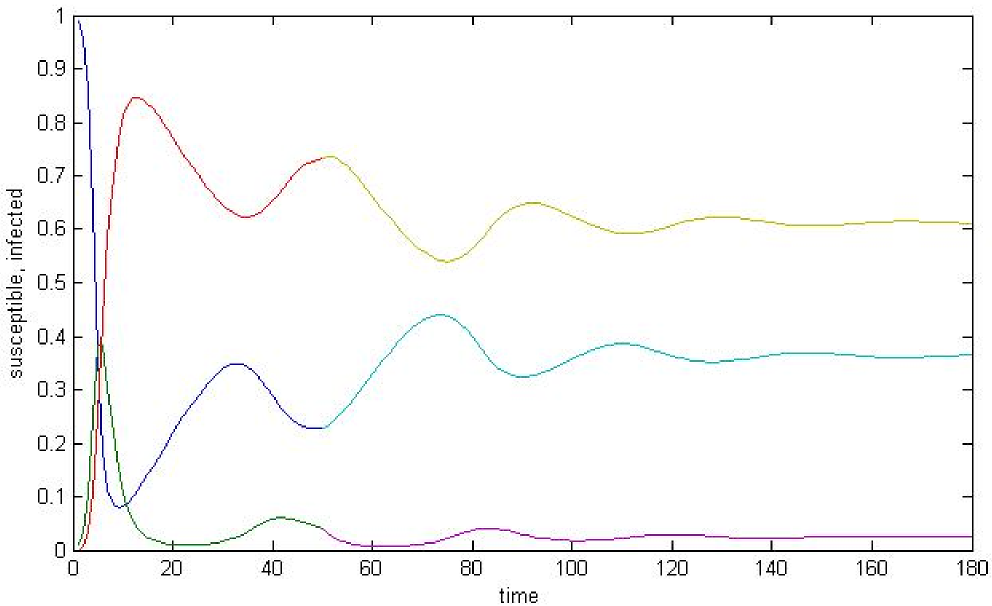
Stopping Drug Use After 20 years



200 years after the disease intro:
S:0.3625
E:0.0016
I:0.0221
T:0.0011
R:0.6127
N:2100

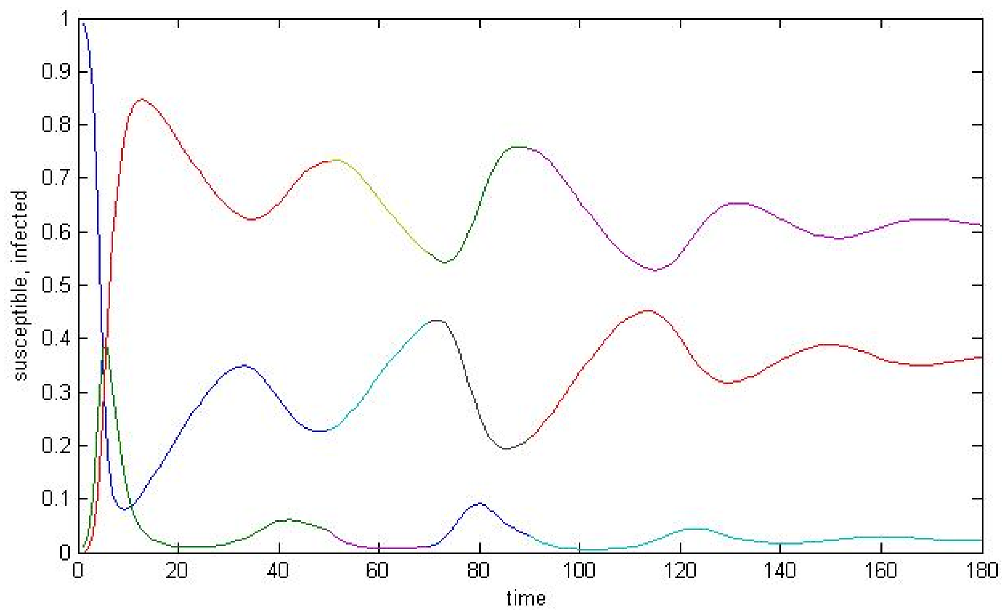
200 years after the disease intro:
S:0.2631
E:0.0018
I:0.0349
T:0
R:0.7002
N:2100





200 years after the disease intro:
S:0.3625
E:0.0016
I:0.0221
T:0.0011
R:0.6127
N:21000

200 years after the disease intro:
S:0.3610
E:0.0017
I:0.0223
T:0.0011
R:0.6132
N:21000



CASE1: CONTINUOUS DRUG USE
CASE2: Intersession for 20 years

- 50 years: Natural Spread
- 150 years: Drug Use

- TOTAL BUDGET
- 246790

- 50 years: Natural Spread
- 20 years: Drug Use
- 20 years: Natural Spread
- 110 years: Drug Use

- TOTAL BUDGET:
- 204610

Conclusion..

- Assumptions:

Birth rate, death rate, recovery rate, contact rate are all constant throughout one's life time

- Modifications

Different disease characteristics might require modifications (for example, what if the disease kills you? What if contact rate depends on whether you are a male or female?)

- Other Factors

Can be even more realistic by adding age/demographic/economic structures into the model.